PROBING LORENTZ SYMMETRY WITH GRAVITATIONALLY COUPLED MATTER

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Methods for obtaining additional sensitivities to Lorentz violation in the fermion sector of the Standard-Model Extension using gravitational couplings are discussed.

1. Introduction

The Standard Model of particle physics together with Einstein's General Relativity provide a remarkably successful description of known phenomena. General Relativity describes gravitation at the classical level, while all other interactions are described down to the quantum level by the Standard Model. However, a single quantum-consistent theory at the Planck scale remains elusive.

Ideally, experimental information would guide the development of the underlying theory; however, directly probing the Planck scale is impractical at present. A feasible alternative is to search for suppressed effects arising from Planck-scale physics in sensitive experiments that can be performed at presently accessible energies. Relativity violations arising from Lorentz-symmetry violation in the underlying theory provide a candidate suppressed effect.[1,2] The Standard-Model Extension (SME) is an effective field theory that describes Lorentz violation at our present energies.[3,4]

A large number of experimental tests of Lorentz symmetry have been performed in the context of the minimal SME. Those test include, in the Minkowski-spacetime limit, experiments with electrons,[5] protons and neutrons,[6] photons,[7] mesons,[8] muons,[9] neutrinos,[10] and the Higgs.[11] The pure-gravity sector has is also being investigated in the post-Newtonian limit.[12,13] Although no compelling experimental evidence for Lorentz violation has been found to date, much remains unexplored. For example, only about half of the coefficients for Lorentz violation involving light and

ordinary matter (protons, neutrons, and electrons) have been investigated experimentally, and other sectors remain nearly unexplored.

In the remainder of this proceedings, a theoretical basis for extending (SME) studies with ordinary matter into the post-Newtonian regime, developed with Alan Kostelecký, will be discussed.[14] The goal of this work is to obtain new sensitivities to Lorentz violation in the fermion sector using couplings to gravity. These couplings introduce new operator structures that provide sensitivities to coefficients for Lorentz violation that are unobservable in Minkowski spacetime.

2. Relativistic Theory

Gravitational effects are incorporated into the SME action in Ref. 4. The general geometric framework assumed is Riemann-Cartan spacetime, which allows for a nonzero torsion tensor $T^{\lambda}_{\ \mu\nu}$ as well as the Riemann curvature tensor $R^{\kappa}_{\ \lambda\mu\nu}$. Due to the need to incorporate spinor fields, the vierbein formalism is adopted. The vierbein also allows one to easily distinguish general coordinate and local Lorentz transformations, a feature convenient in studying Lorentz violation.[4] The spin connection $\omega_{\mu}^{\ ab}$ along with the vierbein $e^{\mu}_{\ a}$ are taken as the fundamental gravitational objects, while the basic non-gravitational fields are the photon A_{μ} and the Dirac fermion ψ.

The minimal-SME action can be expanded in the following way:

$$S = S_G + S_{\psi} + S'. \tag{1}$$

Here, first term, S_G is the action of the pure-gravity sector, which contains the dynamics of the gravitational field and can also contain coefficients for Lorentz violation in that sector.[4,12] The Einstein-Hilbert action of General Relativity is recovered in the limit of zero torsion and Lorentz invariance. The action for the fermion sector is provided by the term S_{ψ} in Eq. (1).

$$S_{\psi} = \int d^4x (\tfrac{1}{2} i e e^{\mu}_{\ a} \overline{\psi} \Gamma^a \overset{\leftrightarrow}{D}_{\mu} \psi - e\overline{\psi} M \psi). \tag{2}$$

Here, Γ^a and M take the form shown in the following definitions.

$$\Gamma^a \equiv \gamma^a - c_{\mu\nu} e^{\nu a} e^{\mu}_{\ b} \gamma^b - d_{\mu\nu} e^{\nu a} e^{\mu}_{\ b} \gamma_5 \gamma^b$$
$$-e_{\mu} e^{\mu a} - i f_{\mu} e^{\mu a} \gamma_5 - \tfrac{1}{2} g_{\lambda\mu\nu} e^{\nu a} e^{\lambda}_{\ b} e^{\mu}_{\ c} \sigma^{bc}. \tag{3}$$

$$M \equiv m + a_{\mu} e^{\mu}_{\ a} \gamma^a + b_{\mu} e^{\mu}_{\ a} \gamma_5 \gamma^a + \tfrac{1}{2} H_{\mu\nu} e^{\mu}_{\ a} e^{\nu}_{\ b} \sigma^{ab}. \tag{4}$$

The symbols a_{μ}, b_{μ}, $c_{\mu\nu}$, $d_{\mu\nu}$, e_{μ}, f_{μ}, $g_{\mu\nu\lambda}$, $H_{\mu\nu}$ are the coefficient fields for Lorentz violation of the minimal fermion sector. In general, they vary with

position and differ for each species of particle. For additional discussion of the fermion-sector action, see Ref. 4.

The final portion, S', of the action (1) contains the dynamics associated with the coefficient fields for Lorentz violation and is responsible for spontaneous breaking of Lorentz symmetry. Through symmetry breaking, the coefficient fields for Lorentz violation are expected to acquire vacuum values. Thus, it is possible to write

$$t_{\lambda\mu\nu\ldots} = \overline{t}_{\lambda\mu\nu\ldots} + \overset{\leftrightarrow}{t}_{\lambda\mu\nu\ldots}, \tag{5}$$

where $t_{\lambda\mu\nu\ldots}$ represents an arbitrary coefficient field for Lorentz violation, $\overline{t}_{\lambda\mu\nu\ldots}$ is the corresponding vacuum value, and $\overset{\leftrightarrow}{t}_{\lambda\mu\nu\ldots}$ is the fluctuations about that vacuum value. Note that a subset of these fluctuations are the massless Nambu-Goldstone modes associated with Lorentz-symmetry breaking.[15] It is possible to develop the necessary tools to analyze fermion experiments in the presence of gravity and Lorentz violation without specifying S'.[14] Those results are summarized here.

The relativistic quantum-mechanical hamiltonian obtained from action (2) provides a first step toward the goal of obtaining experimental access to Lorentz violation. The hamiltonian my be obtained pertrubatively since gravitational and Lorentz-violating effects are small in the regimes of interest. Gravity may be considered perturbatively small in the laboratory and solar-system tests that will be of interest, and Lorentz violation is assumed small since it has not been observed in nature. In what follows, orders in these small quantities will be denoted $O(m, n)$, where m and n are the order of the given term in coefficients for Lorentz violation (vacuum values) and the metric fluctuation respectively.

After incorporating relevant effects to order $m + n = 2$ and making a field redefinition required to define the hamiltonian,[14,16] a hamiltonian of the form,

$$H = H^{(0,0)} + H^{(0,1)} + H^{(1,0)} + H^{(1,1)} + H^{(0,2)}, \tag{6}$$

is found. Here, the Lorentz invariant contributions consist of the conventional Minkowski-spacetime hamiltonian, $H^{(0,0)}$, and the first and second order gravitational corrections denoted $H^{(0,1)}$ and $H^{(0,2)}$ respectively. The first order correction to the hamiltonian due to Lorentz violation, $H^{(1,0)}$, is the same as that found in the flat-spacetime SME.[16] The $O(1,1)$ correction to the hamiltonian is the term of interest since it contains coefficients for Lorentz violation coupled to gravity. It can be written as follows:

$$H^{(1,1)} = H_h^{(1,1)} + H_a^{(1,1)} + H_b^{(1,1)} + \cdots + H_g^{(1,1)} + H_H^{(1,1)}, \tag{7}$$

where $H_h^{(1,1)}$ is the perturbation to the hamiltonian from Lorentz-violating corrections to the metric. The relevant corrections to the metric can be obtained though investigations of the classical limit in Sec. 4. The terms denoted $H_t^{(1,1)}$ are perturbations to the hamiltonian due to Lorentz-violating effects on the test particle at $O(1,1)$. As a sample of $O(1,1)$ effects, $H_a^{(1,1)}$ takes the following form:

$$H_a^{(1,1)} = \overset{\leftrightarrow}{a}_0 - \overline{a}^j h_{j0} + \left[\overset{\leftrightarrow}{a}_j - \tfrac{1}{2}\overline{a}_j h_{00} - \tfrac{1}{2}\overline{a}^k h_{jk}\right]\gamma^0 \gamma^j. \tag{8}$$

The explicit form of the relativistic hamiltonian including all of the coefficients of the minimal fermion sector of the SME, along with additional details of its derivation, can be found in Ref. 14.

3. Non-relativistic Theory

The relativistic hamiltonian above is most useful as a tool to derive the non-relativistic hamiltonian, rather than to analyze experiments directly, because experiments most sensitive to the position operator are non-relativistic. A Foldy-Wouthuysen transformation can be applied to obtain the non-relativistic hamiltonian taking care to maintain the desired order in the small quantities.

After performing the transformation, the non-relativistic hamiltonian, H_{NR}, can be written,

$$H_{\mathrm{NR}} = H_{\mathrm{NR}}^{(0,0)} + H_{\mathrm{NR}}^{(0,1)} + H_{\mathrm{NR}}^{(1,0)} + H_{\mathrm{NR}}^{(1,1)} + H_{\mathrm{NR}}^{(0,2)}. \tag{9}$$

Here, as in the relativistic case, $H_{\mathrm{NR}}^{(0,0)}$ is the conventional Minkowski-spacetime hamiltonian, $H_{\mathrm{NR}}^{(0,1)}$ and $H_{\mathrm{NR}}^{(0,2)}$ contain the leading and sub-leading gravitational corrections, and $H_{\mathrm{NR}}^{(1,0)}$ are the leading corrections due to Lorentz violation, which match Ref. 16. Again, the leading couplings of Lorentz violation to gravitational effects, $H_{\mathrm{NR}}^{(1,1)}$, are the contributions of interest. In a manner analogous to the relativistic case, this term contains contributions from the modified metric fluctuation, written $H_{\mathrm{NR},h}^{(1,1)}$, as well as perturbations from each of the coefficients for the test particle, $H_{\mathrm{NR},t}^{(1,1)}$.

A result of the Foldy-Wouthuysen transformation is that at leading order in the coefficients for Lorentz violation at the non-relativistic level, coefficients a_μ and e_μ always appear in the combination $(a_{\mathrm{eff}})_\mu \equiv a_\mu - m e_\mu$. This result is consistent with those of Ref. 4 indicating that e_μ can be redefined into a_μ at leading order in the coefficients for Lorentz violation via an appropriate field redefinition. As a sample of $O(1,1)$ contributions

to H_{NR}, the correction to the hamiltonian due to a_μ and e_μ can be written

$$H^{(1,1)}_{\mathrm{NR},a_{\mathrm{eff}}} = (\breve{a}_{\mathrm{eff}})_0 + (\overline{a}_{\mathrm{eff}})_k h^{0k} - \frac{1}{m}(\overline{a}_{\mathrm{eff}})^j h_{jk} p^k + \frac{1}{m}\left[(\breve{a}_{\mathrm{eff}})_j - \tfrac{1}{2}(\overline{a}_{\mathrm{eff}})_j h_{00}\right]p^j,$$

(10)

to second order in momentum. Additional details of the derivation of H_{NR}, along with an explicit form for the remaining spin-independent contributions to H_{NR} can be found in Ref. 14.

4. Classical Theory

The classical action can be written as the sum of partial actions

$$S = S_G + S_u + S',$$

(11)

just as in the relativistic case. Here, S_G and S' are as in Eq. (1). The partial action S_u is the point-particle limit of S_ψ appearing in Eq. (1). The classical theory is useful for several applications including obtaining the equations of motion for a classical particle, analyzing non-relativistic quantum-mechanical experiments via the path integral approach, and obtaining the modified metric.

Upon inspection of the non-relativistic hamiltonian discussed in Sec. 3, a point-particle action which corresponds to this hamiltonian can be found. The spin-independent contributions to this action action can be written

$$S_u = \int d\tau \left(-m\sqrt{-(g_{\mu\nu} + 2c_{\mu\nu})u^\mu u^\nu} - (a_{\mathrm{eff}})_\mu u^\mu \right),$$

(12)

where u^μ is the four velocity as usual. The validity of this action has been established here only to within the assumptions made up to the presentation of the non-relativistic hamiltonian, H_{NR}, and in performing calculations with this action one should not exceed the order in small quantities or the order in momentum discussed in the last section. In addition to the match between the classical action above and the non-relativistic hamiltonian, the fact that the dispersion relation generated by the classical action matches the relativistic theory confirms the validity of this action. Note also that the $c_{\mu\nu}$ contributions to this action have been discussed previously in the context of work done in the photon sector.[17] After some consideration,[14] this action can be extended to address the case in which particles are bound within macroscopic matter as well.

5. Experimental Tests

With the theory developed above, experiments in any regime, from relativistic quantum mechanics to classical mechanics, can be analyzed, pro-

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vided the contributions from S' are known. These contributions can be established directly within a model of spontaneous Lorentz violation or determined for a large class of models by examining the general form of the contributions along with the constraints available from conservation laws.

Upon a general analysis, effects are found in a number of experiments. These effects include annual and sidereal variations in the newtonian gravitational acceleration as well as variations in the gravitational force based on the proton, neutron, electron content of the bodies involved. These effects lead to signals in gravimeter tests as well as in some experiments designed to test the weak equivalence principle. These tests are described in detail in Ref. 14 and will provide the first direct sensitivities to the $(\overline{a}_{\text{eff}})_\mu$ coefficients for the proton, neutron, and electron.

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